Comment on "Coherent Ratchets in Driven Bose-Einstein Condensates"

Creffield and Sols (henceforth CS) [1] recently reported a finite, directed time-averaged ratchet current, for *non-interacting* quantum particles in a potential $V(x,t) = KV(x)f(t)$ with time-periodic driving $f(t) = f(t+T)$, even when time-reversal symmetry holds, as depicted with the solid line in Fig. 3 in [1]. CS chose $V(x) = \sin(x) + \alpha\sin(2x)$, $f(t) = \sin(t) + \beta\sin(2t)$ ($\beta = 0$ in their Fig. 3) and the initial condition $\Psi(x,0) = 1/\sqrt{2\pi}$. As we will explain in the following, this result is incorrect, that is, time-reversal symmetry implies a vanishing ratchet current.

The asymptotic time averaged current (TAC) is given by $J = \lim_{\tau\to\infty} J(\tau)$ where $J(\tau) = \tau^{-1}\int_0^\tau I(t)dt$. $I(t)$ is given by $I(t) = -i\int_{-\infty}^{\infty} dx\ \Psi^*(x,t)\frac{\partial\Psi(x,t)}{\partial x}$. Given the periodicity of the driving, $f(t) = f(t+T)$, one may analyze the evolution in terms of the system's Floquet states. The asymptotic TAC is then given by [2]

$$J = \sum_\alpha |C_\alpha|^2 \langle\langle\psi_\alpha|\hat{p}|\psi_\alpha\rangle\rangle_T = \sum_\alpha |C_\alpha|^2 \langle v_\alpha(t)\rangle_T, \quad (1)$$

where ψ_α are the Floquet eigen-states (FES), $\psi_\alpha(t+T) = \psi_\alpha(t)$, the coefficients C_α are such that $\Psi(x,0) = \sum_\alpha C_\alpha\psi_\alpha(x,0)$, $v_\alpha(t) = -i\int_{-\infty}^{\infty} dx\ \psi_\alpha^*(x,t)\frac{\partial\psi_\alpha(x,t)}{\partial x}$ is the instantaneous velocity of the Floquet state, and $\langle ...\rangle_T$ denotes the average in time over the period T. The TAC for each FES vanishes identically if $f(t_s+t) = f(t_s-t)$ for some t_s, because $v_\alpha(t_s+t) = -v_\alpha(t_s-t)$, and therefore $\langle v_\alpha(t)\rangle_T = 0$ [2]. Given that J is the weighted sum (1), it follows that $J = 0$ for $\beta = 0$ because $\sin(\pi/2+t) = \sin(\pi/2-t)$. Since the parameter K does not change the symmetries of the system, and given that the time-reversal symmetry implies a vanishing TAC, we conclude that no asymptotic directed transport occurs for any value of this parameter. CS used the stroboscopic current, $J_s(t_p,m) = \dfrac{1}{m+1}\displaystyle\sum_{n=0}^{m} I(t_p+nT)$. Their asymptotic stroboscopic current is given by [2]

$$J_s(t_p) = \sum_\alpha |C_\alpha|^2 v_\alpha(t_p) = J_s(t_p+T), \quad (2)$$

where $v_\alpha(t)$ are periodic functions, $v_\alpha(t+T) = v_\alpha(t)$. Since even in the case of time-reversal symmetry instantaneous velocities are nonzero, $v_\alpha(t_p) \neq 0$, the current (2) acquires a nonzero value, which depends on the arbitrary choice of the measurement time $t_p \in [0,T)$.

Motion is a continuous process and attempts to describe it in terms of stroboscopic characteristics only may lead to wrong physical conclusions. The harmonic oscillator constitutes a good example: Its particle velocity is $v(t) = v_0\sin[\omega(t-t_p)]$ and, depending on t_p, the asymptotic stroboscopic averaged velocity $v_s(t_p)$ may take any value within the interval $[-v_0, v_0]$, although no directed transport occurs. We numerically verified the above conclusions by performing an integration of the Schrödinger equation with the same parameters as in Fig. 3 of [1]. We used two independent methods [2, 3]. The so obtained results do coincide and are depicted in our Fig. 1. For $\beta = 0$ we numerically obtain virtually zero current for all values of α, the thick (blue) solid line. The amplitude of small fluctuations away from zero decrease systematically upon increasing the overall integration time τ, see inset in Fig. 1. These findings are therefore in full agreement with the symmetry analysis [2]. In the contrast, the stroboscopic current used in Ref. [1] remains finite forever, approaching values predicted by (2). Moreover, the above symmetry analysis is not in contrast with Ref. [3], where the atom-atom interactions obey time reversal symmetry.

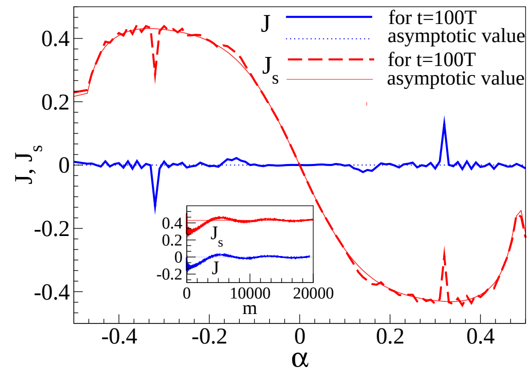

FIG. 1: (Color online) $J(t)$ and the stroboscopic current $J(0,m)$ as functions of α: for $t = mT$, where $m = 100$, thick blue solid line and thick red dashed line correspondingly; and their asymptotic values, J, Eq. (1), (thin blue dotted line) and J_s, Eq. (2)(thin red solid line). Here $K = 2.4$ and $\beta = 0$. Inset: Dependence of $J(t = mT)$ (blue line) and $J_s(t_p = 0, m)$ (red line) on m at $\alpha = -0.32$. The thin lines are given by (1) and (2), respectively.

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[1] C. E. Creffield and F. Sols, Phys. Rev. Lett. **103**, 200601 (2009).

[2] S. Denisov *et al.*, Phys. Rev. A **75**, 063424 (2007).

[3] D. Poletti *et al.*, Phys. Rev. Lett. **102**, 130604 (2009).

Supplementary material for our published comment [G. Benenti *et al.*, Phys. Rev. Lett. **104**, 228901 (2010)]. In view of the published "Reply" by CS [C. E. Creffield and F. Sols, Phys. Rev. Lett. **104**, 228902 (2010)], which we comment-authors could not referee/act upon, we herewith like to elucidate some of their erroneous claims stated therein.

In their Reply, Creffield and Sols (henceforth CS) accuse us of making false assertions and reading without sufficient care their paper on the basis of an estimate developed therein for *interacting* particles: Consequently, this discussion does not apply in the context of our Comment which deals with the *non-interacting* (N=1) case. Furthermore, such an estimate, if extended to the non-interacting case, would contradict the results of the stroboscopic average used by CS. Finally, and most importantly, the stroboscopic average cannot be used to detect directed transport and, in the present model study, provides wrong results both asymptotically and for the experimentally relevant time scales: First, the stroboscopic average gives in presence "of no temporal asymmetry" (see in abstract and in Fig. 3 in the original PRL by CS) a finite, non-vanishing asymptotic stroboscopic current instead of a vanishing zero asymptotic ratchet current. Second, the stroboscopic average fails, even at short times, to provide an acceptable approximation of the current: Even the sign of the transporting ratchet current can be wrong, see Fig. 1 of our Comment above and its inset with the blue and red line. Let us discuss in more detail those erroneous statements by CS in their PRL-Reply:

1.

Us being accused: – in Line 4: "Their assertion is false". This sentence is wrong: We authors of the comment have not stated that the ratchet current for the time-symmetric case persists indefinitely but rather we referred to their explicit claim that (we here carefully quote from their PRL-abstract) "for strong driving, the behaviour becomes chaotic and the resulting effective irreversibility means that it is unnecessary to explicitly break time symmetry." Then continuing and stating that "Spatial asymmetry alone is then sufficient to produce the ratchet effect, even in the absence of interaction...."

First, we would like to stress again that we are only discussing the non-interacting case: For this case the number of atoms is $N = 1$. So the analysis of the decay time presented at page 4 of CS's Phys. Rev. Lett. **103**, 200601 (2009): (carefully quoting from their PRL) "This current should remain stable over time scales comparable to the systems's Ehrenfest time, which scales logarithmically with number of atoms in the condensate. For a condensate of 10^5 atoms we estimate this time to be of the order of 50 driving periods". This statement thus cannot make sense for the non-interacting case; using the estimate by Shepelyansky and co-workers, Ref. [20] in their PRL and now Ref. [3] in their PRL-Reply, would yield $\ln\ N = \ln\ 1 = 0$. It seems that the only thing one can derive from this argument is that in the non-interacting case the decay time is 0; so what is the meaning of the current values depicted with their Fig. 3 in CS-PRL for 100 driving periods, and reproduced in our Fig. 1 with J_s for $t = 100T$?

Clearly there is something wrong here in their estimation of decay time for the noninteracting case.

2.

It is also very disturbing that the authors in their Reply still fail to clarify the fact, – as proven by us numerically with our Fig. 1, that they in fact calculated in their PRL the stroboscopic current, rather than the time-continuous transporting ratchet current; the latter obeying rigorously the symmetry analysis and describing the physical transport features of ratcheted atoms. This stroboscopic current then yields non-decaying, finite values while the (time-continuous averaged) transporting, physical ratchet current is vanishing for all α-values, see Fig. 1 in our Comment. Moreover, by computing the stroboscopic current one cannot decide whether the atoms are acquiring a finite average current moving in one direction, or are just oscillating forth and back. This insight is of relevance for experimental realizations.

3.

Their polemical statement on Line 8, reading: "Unfortunately Benenti *et al.* appear not to have read our paper with sufficient care". This constitutes a polemic statement which is not acceptable and is insulting to us authors. No reader can defer from the original CS-PRL anything else than the wrong result of a finite ratchet current (depicted with Fig. 3 for $K = 2.4$ and $g = 0$) for a time-reversal symmetric situation.

4.

Their claim: "This yields the *approximate* formula in Eq. 1 of Ref. [1], in which solely the diagonal terms are retained, effectively ..... ". First, note that in this context we cite in our Comment the prior work [2]; Denisov *et al.*, Phys. Rev. A **75**: 063424 (2007). We must stress that our Eq. (1) is not approximate but simply exact for the situation under consideration: A time-dependent driven, nonlinear periodic potential. This situation yields a quasienergy spectrum with avoided crossings which for specific parameters may indeed be very small; but after all these are still avoided. This implies that our formula Eq.(1) is exact because it refers to the time-asymptotic, i.e. $t \to \infty$ result of the continuous time averaged coherent quantum ratchet current. The approach towards "zero" is a matter of time scales, but it occurs asymptotically. Upon varying a single parameter only, i.e. α in Fig. 3, no such mathematical exact crossing occurs. We stress that CS also fail to identify such a case of an exact crossing for the given situation under discussion. So there is no justification to discredit our result in (1) by highlighting it (in italics) as an "approximate" one.

In this same context the following statements in their present Reply are false when CS write: "......or will only decay extremely slowly (when degeneracy is approximate). Although the analysis of Benenti *et al.* cannot describe this situation, our approach would simply yield a longer Ehrenfest time in such cases." This is false: In particular, as stated clearly in our Comment, our Eq. (1) provides the asymptotic values; i.e. our Eq. (1) (clearly not being a function of "t") is not meant to be applied (wrongly) to a finite, transient time "t". It therefore does very well cover also the cases of small avoided crossings, see our inset in our Fig 1 for $\alpha = 0.32$. Likewise in their concluding sentence of this paragraph: "The conclusion of Benenti *et al.* that "no asymptotic transport occurs for any value of K" is thus not generally correct - it depends on the detailed form of the quasienergy spectrum" does not hold up to inspection. CS cannot provide a single case for this driven

system under consideration in which there occurs an exact degeneracy.

5.

CS in their Reply camouflage their erroneous reasoning inherent in their PRL with the use of the stroboscopic average, which additionally they fail to admit that the stroboscopic average depends on the time "t_p" which "clocks" the asymptotic, periodically varying stroboscopic average, see our Eq. (2) in our Comment. Apparently they have chosen this "clock time" as $t_p = 0$ with their Fig. 3 in their PRL. Note in this context the close agreement of their Fig. 3 with our Fig. 1. This choice of "$t_p = 0$" is again not specified in Fig. 2 of their supplementary material.

6.

Overall, CS in their Reply now focus on the transient behaviour of the currents near narrow avoided crossings which occur for both, at $\alpha = \pm 0.32$ and $\alpha = \pm 0.15$ (with $\beta = 0$) (their Supplementary material). We respectfully contest their shift in issue in their present Reply from the issue addressed with our Comment in virtue of the erroneous "ratchet current produced for system with no temporal asymmetry" (quoted from the caption for Fig. 3 in CS-PRL) and their issue of the slow transients behaviour occurring near such narrow avoided crossings. The latter issue is not mentioned once in their PRL. Moreover, this discussion is neither original nor new: It cannot have escaped the attention of CS that this very issue of narrow avoided crossings and corresponding slow transient behaviour has been discussed by some of us before with the cited Ref. [2] (Phys. Rev. A **75**: 063424 (2007)); please note therein Fig. 2 (narrow avoided crossings) and the transient current behaviour depicted with Fig. 9 therein.

7.

Finally, we do agree with CS when they state in the "Reply" that our given example of a harmonic oscillator is trivial. The latter illustrates, however, in a simple and most transparent way the source for erroneous reasoning that occurs with a stroboscopic averaging procedure. In particular, it shows why a periodic function like the asymptotic stroboscopic current, $J_s(t_p) = ... = J_s(t_p+T)$, our Eq. (2), cannot yield the physically relevant quantum ratchet current.